

Find all the four second order partial derivatives (Problems 23 - 26).

23. e^{x-y}

24. $\frac{x+y}{x-y}$

25. e^{xy}

26. $\tan(\arctan x + \arctan y)$

In Problems 27 - 32 verify that $f_{xy} = f_{yx}$

27. $f(x, y) = e^{xy} \cos(bx + c)$

28. $f(x, y) = \ln(e^x + e^y)$

29. $f(x, y) = \ln\left(\frac{x^2 + y^2}{xy}\right)$

30. $f(x, y) = x^y + y^x$

31. $f(x, y) = x \sin xy + y \cos xy$

32. $f(x, y) = \frac{xy}{\sqrt{1 + x^2 + y^2}}$

Show that each of the following functions satisfies Laplace's equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$. (Problem 33 - 36):

33. $f(x, y) = \sin x \sinh y$

34. $f(x, y) = e^{-x} \cos y$

35. $f(x, y) = \ln \sqrt{x^2 + y^2}$

36. $f(x, y) = \arctan\left(\frac{2xy}{x^2 - y^2}\right)$

37. If $f(x, y) = x^2 \arctan\left(\frac{y}{x}\right) - y^2 \arctan\left(\frac{x}{y}\right)$, show that

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}.$$

38. If $f(x, y) = \frac{x^2 + y^2}{x + y}$, prove that

$$(f_x - f_y)^2 = 4(1 - f_x - f_y)$$

39. Show that the function $f(x, y) = \sin xy$ satisfies the differential equation $x^2 f_{xx} - y^2 f_{yy} = 0$.

40. Let $f(x, y) = \begin{cases} x^2 \arctan\left(\frac{y}{x}\right) - y^2 \arctan\left(\frac{x}{y}\right) & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$

Show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$.

41. (i) Let $f(x, y, z) = x^3 + 3yz + \sin xyz$. Prove that $f_{xyz} = f_{zxy}$.

(ii) If $f(x, y, z, w) = \frac{xy}{z + w}$, show that $f_{xyzw} = \frac{2}{(x + w)^3}$.

In Problems 42 - 45, find $\frac{dy}{dx}$ by using partial derivatives:

42. $y^2 + x^2 y + ax^4 = 0$

43. $3x^2 - y^2 + x^3 = 0$

44. $x^2 + xy + y^2 + ax + by = 0$

45. $x^3 + x^2 + xy^2 + \sin y = 0$